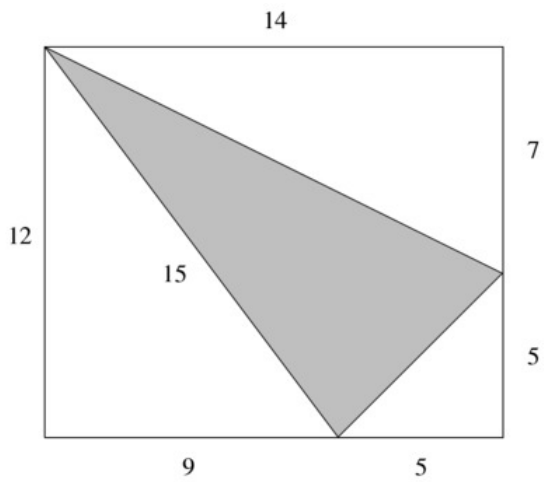
Illustrative Mathematics

8.G Applying the Pythagorean Theorem in a mathematical context

Alignment 1: 8.G.B

Three right triangles surround a shaded triangle; together they form a rectangle measuring 12 units by 14 units. The figure below shows some of the dimensions but is not drawn to scale.



Is the shaded triangle a right triangle? Provide a proof for your answer.

Solution: I. Proof by contradiction

Using the Pythagorean Theorem, we can find the hypotenuse c of the smallest right triangle.

$$5^2 + 5^2 = c^2$$

so

$$c = \sqrt{5^2 + 5^2} = 5\sqrt{2}.$$

Similarly, the hypotenuse of the right triangle with side lengths 7 and 14 is

$$\sqrt{7^2 + 14^2} = 7\sqrt{5}.$$

If the shaded triangle is a right triangle, then the side-lengths must satisfy the Pythagorean Theorem. Since $7\sqrt{5}$ is the longest of the three sides, it would be the hypotenuse, so if this is a right triangle, then the following equation must be true:

$$(5\sqrt{2})^2 + 15^2 = (7\sqrt{5})^2.$$

However, looking at the left-hand side, we find that

$$(5\sqrt{2})^2 + 15^2 = 50 + 225 = 275$$

and looking at the right hand side, we find that

$$(7\sqrt{5})^2 = 245,$$

and the equation is not true. So the shaded triangle is not a right triangle.

Solution: II. Simpler version of solution I.

In solution I, it is not necessary to take square roots to find the hypotenuses, since it is only the squares of the hypotenuses that are need to verify that the shaded triangle is not a right triangle.

Using the Pythagorean Theorem, we can find the square of the hypotenuse of the smallest right triangle.

$$5^2 + 5^2 = 50.$$

Similarly, the square of the hypotenuse of the right triangle with side lengths 7 and 14 is

$$7^2 + 14^2 = 245.$$

The square of the third side of the shaded triangle is

$$15^2 = 225.$$

If the shaded triangle is a right triangle, then squares of the side lengths must satisfy Pythagoras's theorem. We have found that those squares are 50, 245, and 225. The largest of these is 245, so the other two must add up to 245. But $50 + 225 = 275 \neq 245$. So the shaded triangle is not a right triangle.

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