

Triangle Angle Theorems

Essential question: *What can you conclude about the measures of the angles of a triangle?*

COMMON CORE

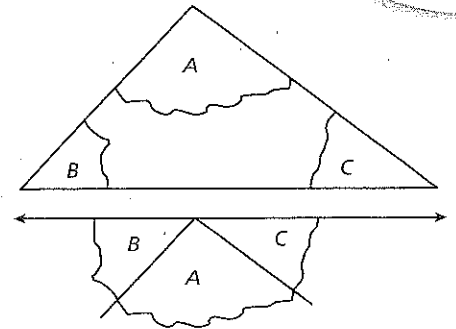
CC.8.G.5

1 EXPLORE Sum of the Angle Measures in a Triangle

There is a special relationship between the measures of the interior angles of a triangle.

- A Draw a triangle and cut it out. Label the angles A , B , and C .
- B Tear off each "corner" of the triangle. Each corner includes the vertex of one angle of the triangle.
- C Arrange the vertices of the angle around a point so that none of your corners overlap and there are no gaps between them.
- D What do you notice about how the angles fit together around a point?

- E What is the measure of a straight angle? _____
- F Describe the relationship among the measures of the angles of $\triangle ABC$ in words.

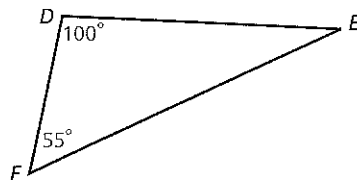


The Triangle Sum Theorem states that for $\triangle ABC$,
 $m\angle A + m\angle B + m\angle C =$ _____.

TRY THIS!

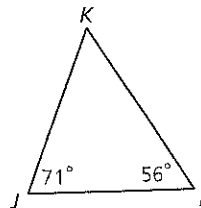
Find the missing angle measure.

1a.



$$\begin{aligned}
 55^\circ + & \quad + m\angle E = 180^\circ \\
 155^\circ + & m\angle E = 180^\circ \\
 155^\circ - & + m\angle E = 180^\circ - \\
 & m\angle E =
 \end{aligned}$$

1b.



You can use your knowledge of parallel lines intersected by a transversal to informally justify the Triangle Sum Theorem.

2 EXPLORE Justifying the Triangle Sum Theorem

Follow the steps to informally prove the Triangle Sum Theorem. You should draw each step on your own paper. The figures below are provided for you to check your work.

A Draw a triangle and label the angles as $\angle 1$, $\angle 2$, and $\angle 3$ as shown.

B Draw line a through the base of the triangle.

C The Parallel Postulate states that through a point not on a line ℓ , there is exactly one line parallel to line ℓ . Draw line b parallel to line a , through the vertex opposite the base of the triangle.

D Extend the other sides of the triangles to form transversal s and transversal t . Transversals s and t intersect parallel lines a and b .

E Label the angles formed by line b and the transversals as $\angle 4$ and $\angle 5$.

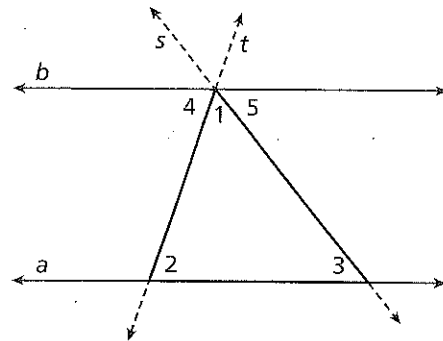
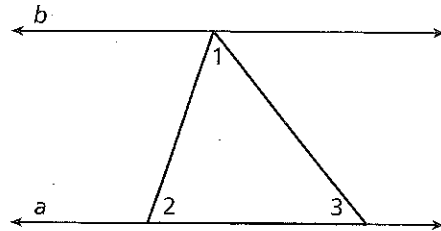
F $\angle 4$ and _____ are alternate interior angles, so they are _____. Label $\angle 4$ with the number of the angle it is congruent to.

G $\angle 5$ and _____ are alternate interior angles, so they are _____. Label $\angle 5$ with the number of the angle it is congruent to.

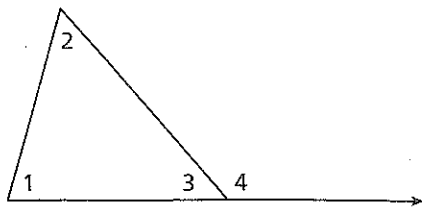
H The three angles that lie along line b at the vertex of the triangle are $\angle 1$ and two angles that have the same measures as $\angle 2$ and $\angle 3$. Because these three angles lie along a line, the sum of their measures is _____.

$$m\angle 1 + m\angle 2 + m\angle 3 = \underline{\hspace{2cm}}$$

The sum of the angle measures in a triangle is always _____.



An **interior angle** of a triangle is formed by two sides of the triangle. An **exterior angle** of a triangle is formed by one side of the triangle and the extension of an adjacent side. Each exterior angle has two *remote interior angles*. A **remote interior angle** is an interior angle that is not adjacent to the exterior angle.



- $\angle 1$, $\angle 2$, and $\angle 3$ are interior angles.
- $\angle 4$ is an exterior angle.
- $\angle 1$ and $\angle 2$ are remote interior angles to $\angle 4$.

3 EXPLORE Exterior Angles and Remote Interior Angles

There is a special relationship between the measure of an exterior angle and the measures of its remote interior angles.

A Extend the base of the triangle and label the exterior angle as $\angle 4$.

B The Triangle Sum Theorem states:

$$m\angle 1 + m\angle 2 + m\angle 3 = \underline{\hspace{2cm}}$$

C $\angle 3$ and $\angle 4$ form a $\underline{\hspace{4cm}}$,

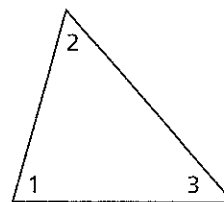
so $m\angle 3 + m\angle 4 = \underline{\hspace{2cm}}$.

D Use the equations in **B** and **C** to complete the following equation:

$$m\angle 1 + m\angle 2 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} + m\angle 4$$

E Use properties of equality to simplify the equation in **D** :

$\underline{\hspace{10cm}}$



Key The Exterior Angle Theorem states that the measure of an $\underline{\hspace{4cm}}$ angle is equal to the sum of its $\underline{\hspace{4cm}}$ angles.

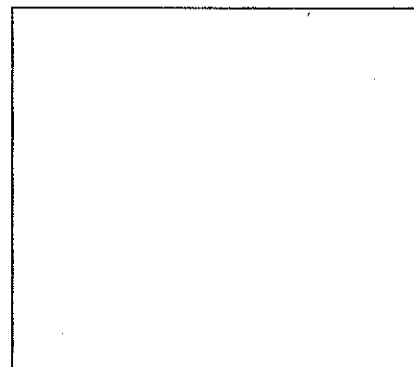
REFLECT

3a. Sketch a triangle and draw all of its exterior angles. How many exterior angles does a triangle have at each vertex?

$\underline{\hspace{10cm}}$

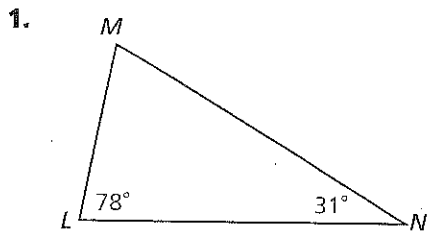
3b. How many total exterior angles does a triangle have?

$\underline{\hspace{10cm}}$

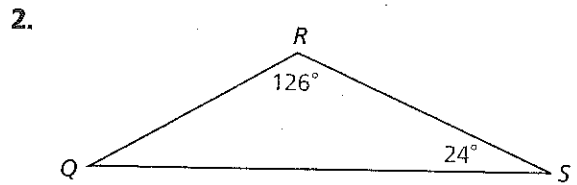


PRACTICE

Find the missing angle measure.

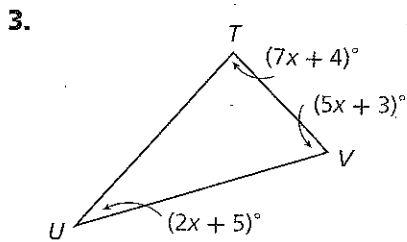


$m\angle M =$ _____



$m\angle Q =$ _____

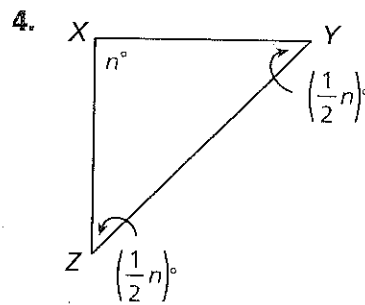
Use the Triangle Sum Theorem to find the measure of each angle in degrees.



$m\angle T =$ _____

$m\angle U =$ _____

$m\angle V =$ _____

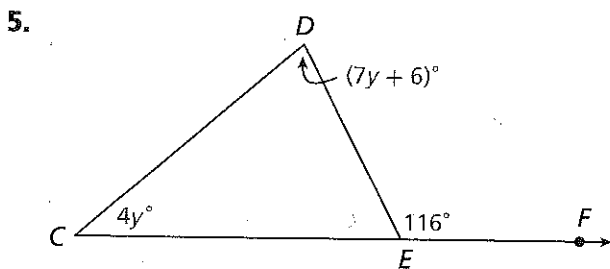


$m\angle X =$ _____

$m\angle Y =$ _____

$m\angle Z =$ _____

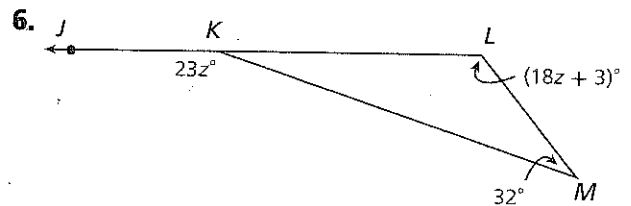
Use the Exterior Angles Theorem to find the measure of each angle in degrees.



$m\angle C =$ _____

$m\angle D =$ _____

$m\angle DEC =$ _____



$m\angle L =$ _____

$m\angle MKL =$ _____

$m\angle MKJ =$ _____